

The Gauge/Gravity Correspondence for Non-Supersymmetric Theories

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Abstract: We review the string construction of the “orientifold field theories” and we show that for these theories the gauge/gravity correspondence is only valid for a large number of colours.

1 Introduction

In recent years it has been possible to get a lot of perturbative and non-perturbative information about less supersymmetric and non-conformal four-dimensional gauge theories living on the world-volume of wrapped and fractional D branes by using their corresponding classical solutions of the equations of motion of the low-energy string effective action ¹. These equations contain not only the supergravity fields present in the bulk ten-dimensional action but also boundary terms corresponding to the location of the branes. It turns out that in general the classical solution develops a naked singularity of the repulsion type at short distances from the branes. On the other hand at these distances it does not provide anymore a reliable description of the branes because of the presence of an enhançon located at distances slightly higher than the naked singularity [6]. The enhançon radius corresponds in the gauge theory living on the branes to the dynamically generated scale Λ_{QCD} . Then, since short distances in supergravity correspond to large distances in the gauge theory, as implied by holography, the presence of the enhançon does not allow to get information about the large distance behaviour of the gauge theory living on the D branes. Above the radius of the enhançon instead the classical solution provides a good description of the branes and therefore it can be used to get information on the perturbative behaviour of the gauge theory. This is why the perturbative behaviour of $\mathcal{N} = 2$ super Yang-Mills with gauge group $SU(N)$ has been obtained from the classical solution corresponding to a bunch of N fractional D3 branes of the orbifold $\mathbb{C}^2/\mathbb{Z}_2$ [7, 8, 9]. On the other hand, from the point of view of string theory, the one-loop perturbative behaviour of the gauge theory living on the world-volume of a system of D branes can be obtained from the annulus diagram by performing the field theory limit in the open string channel.

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¹For general reviews of various approaches see Ref.s [1, 2, 3, 4, 5].

This is a consequence of the fact that this limit has the effect of allowing only the massless open string states to circulate in the loop. But, since the classical solution involves only the massless closed string states, the question is then: why is the contribution of the massless open string states equivalent to that of the massless closed string states? The previous results seem to imply that the contribution of the massless open string states in the annulus diagram is transformed under open/closed string duality into that of the massless closed string states. This has been shown [10] in fact to happen for the fractional D3 branes of the orbifolds $\mathbb{C}^2/\mathbb{Z}_2$ and $\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ of type IIB that describe respectively $\mathcal{N} = 2$ and $\mathcal{N} = 1$ super Yang-Mills. In particular, in these cases it has been shown that the threshold corrections due to the massive string states are identically vanishing. These properties seem to be valid also for fractional D3 branes on the conifold and for D branes wrapped on Calabi-Yau two-cycles where an explicit string analysis is not possible. Can these properties be extended to non-supersymmetric theories? Recently the so called “orientifold field theories” have been studied. They have the property of reducing to a supersymmetric one in the large number N of colours. A string description of some of them has been provided in Ref. [11] where it has been also analyzed if the properties of the supersymmetric theories discussed above can be extended to them. In this talk we will review these results. In Sect. (2) we summarize what happens in the case of the supersymmetric $\mathcal{N} = 2$ theories. In Sect. (3) we give a string construction of some orientifold field theories and in Sect. (4) we analyze if the gauge/gravity correspondence is valid for them.

2 Gauge/gravity correspondence for supersymmetric theories

Let us consider N fractional D3 branes of type IIB string theory on the orbifold $\mathbb{R}^{1,5} \times \mathbb{C}^2/\mathbb{Z}_2$ having their world-volume along the directions 0, 1, 2, 3, while the non-trivial element of \mathbb{Z}_2 acts as $x^i \rightarrow -x^i$ along the directions $i = 6, 7, 8, 9$. The fractional D3 branes are coupled to the untwisted massless closed string excitations corresponding to the graviton, the dilaton and the R-R 4-form C_4 and to the twisted ones corresponding to $B_2 = \omega_2 b$ and $C_2 = \omega_2 c$, where ω_2 is the volume-form of the vanishing 2-cycle located at the orbifold fixed point and such that $\int_{C_2} \omega_2 = 1$. The classical supergravity solution corresponding to a fractional D3 brane is given by ($\alpha, \beta = 0 \dots 3, i, j = 4 \dots 9$):

$$ds^2 = H^{-1/2} \eta_{\alpha\beta} dx^\alpha dx^\beta + H^{1/2} \delta_{ij} dx^i dx^j , \quad (1)$$

$$\tilde{F}_{(5)} = d(H^{-1} dx^0 \wedge \dots \wedge dx^3) + {}^*d(H^{-1} dx^0 \wedge \dots \wedge dx^3) , \quad (2)$$

$$c = -N(2\pi\sqrt{\alpha'})^2 \frac{g_s}{\pi} \theta \quad ; \quad b = N(2\pi\sqrt{\alpha'})^2 \frac{g_s}{\pi} \log(\rho/\epsilon) \quad (3)$$

where $z \equiv x^4 + ix^5 = \rho e^{i\theta}$ and H is given in Ref. [8].

The supergravity solution provides a non-trivial information on the four-dimensional gauge theory living on the world-volume of N fractional branes, namely $SU(N)$ $\mathcal{N} = 2$ super Yang-Mills. This can be obtained by considering the world-volume theory (Dirac-Born-Infeld + Wess-Zumino term) of a fractional D3 brane. In the $\alpha' \rightarrow 0$ limit, keeping fixed the scalar field $\phi = \frac{x_4+ix_5}{2\pi\sqrt{2}\alpha'}$ of the gauge multiplet of $\mathcal{N} = 2$ super Yang-Mills, one obtains

$$S_{\text{YM}} = -\frac{1}{g_{\text{YM}}^2} \int d^4x \left\{ \frac{1}{4} F_{\alpha\beta}^a F_a^{\alpha\beta} + \frac{1}{2} \partial_\alpha \bar{\phi} \partial^\alpha \phi \right\} + \frac{\theta_{\text{YM}}}{32\pi^2} \int d^4x F_{\alpha\beta}^a \tilde{F}_a^{\alpha\beta} \quad (4)$$

where the Yang-Mills coupling constant is given by:

$$\frac{1}{g_{\text{YM}}^2} = \tau_5 \frac{(2\pi\alpha')^2}{2} \int_{C_2} B_2 = \frac{b}{16\pi^3\alpha' g_s} = \frac{1}{8\pi g_s} + \frac{2N}{8\pi^2} \log \frac{\rho}{\epsilon} \quad (5)$$

and the θ -angle by ($\tau_5 = [g_s \sqrt{\alpha'} (2\pi \sqrt{\alpha'})^5]^{-1}$):

$$\theta_{\text{YM}} = \tau_5 (2\pi\alpha')^2 (2\pi)^2 \int_{C_2} C_2 = \frac{c}{2\pi\alpha' g_s} = -2N \theta. \quad (6)$$

Since the supergravity coordinate z corresponds to the complex scalar field ϕ of $\mathcal{N} = 2$ super Yang-Mills and we know how the scale and chiral anomalous transformations act on it, we can infer how they act on z :

$$\phi \rightarrow \mu e^{2i\alpha} \phi \iff z \rightarrow \mu e^{2i\alpha} z. \quad (7)$$

Acting with the previous transformations in Eq.s (5) and (6) we get:

$$\frac{1}{g_{\text{YM}}^2} \rightarrow \frac{1}{g_{\text{YM}}^2} + \frac{2N}{8\pi^2} \log \mu \quad ; \quad \theta_{\text{YM}} \rightarrow \theta_{\text{YM}} - 4N\alpha. \quad (8)$$

These equations reproduce the β -function and the chiral anomaly of $SU(N)$ $\mathcal{N} = 2$ super Yang-Mills.

We will now show that the fact that the supergravity solution reproduces the perturbative properties of the gauge theory living in the world-volume of N fractional D3 branes is a consequence of the absence of threshold corrections to the running gauge coupling constant and the θ -angle. This can be seen by computing the annulus diagram describing the interaction between a set of N fractional D3 branes and a fractional D3 brane having an external gauge field in its world-volume and located at a distance ρ from the other branes. In the case of the orbifold $\mathbb{R}^{1,5} \times \mathbb{C}^2/\mathbb{Z}_2$ the annulus diagram has two contributions: one from the untwisted and the other from the twisted sector:

$$Z = \int_0^\infty \frac{d\tau}{\tau} Tr_{\text{NS-R}} \left[\left(\frac{1+h}{2} \right) (-1)^{G_{bc}} P_{GSO} e^{-2\pi\tau L_0} \right] = Z_e + Z_h, \quad (9)$$

but only the twisted sector gives a non-zero contribution to the anomalies. One can extract from the annulus diagram the contribution to the gauge coupling constant and to the θ vacuum. It can be extracted either from the open or from the closed string channel. From the open string channel one gets:

$$\left[-\frac{1}{4} \int d^4x F^2 \right] \left[-\frac{N}{8\pi^2} \int_{1/(\alpha'\Lambda^2)}^\infty \frac{d\tau}{\tau} e^{-\frac{\rho^2\tau}{2\pi\alpha'}} \right] - iN \left[\frac{1}{32\pi^2} \int d^4x F \cdot \tilde{F} \right] \int_0^\infty \frac{d\tau}{\tau} e^{-\frac{\rho^2\tau}{2\pi\alpha'}}. \quad (10)$$

Notice that the sign in front of the topological term is opposite to the sign appearing in Ref.s [10, 11] because in these papers we have used a different GSO projection in the R sector. From Eq. (10) we see that only the massless open string states contribute to the gauge coupling constants and their contribution under open/closed string duality goes exactly into that of the massless closed string states that appear in the supergravity solution. The massive states in both channels give no contribution and this implies the absence of threshold corrections. This is basically the reason why one can read the perturbative behaviour of $\mathcal{N} = 2$ super Yang-Mills from the classical supergravity solution. The expressions that we get from Eq. (10) are, however, divergent already at

the string level and in addition the θ -angle is imaginary. In Ref. [11] we have discussed how to cure the two problems by introducing a complex cut-off. Here we give just the final results:

$$\frac{1}{g_{YM}^2} = \frac{1}{g_{YM}^2(\Lambda)} - \frac{N}{8\pi^2} \frac{1}{2} [I(z) + I(\bar{z})] = \frac{1}{g_{YM}^2(\Lambda)} + \frac{N}{8\pi^2} \log \frac{\rho^2}{2\pi(\alpha')^2 \Lambda^2} \quad (11)$$

and

$$\theta_{YM} = -iN \cdot \frac{1}{2} [I(z) - I(\bar{z})] = -2N\theta \quad (12)$$

where

$$I(z) \equiv \int_{1/(e^{-i\theta}\Lambda)^2}^{\infty} \frac{d\sigma}{\sigma} e^{-\frac{\rho^2\sigma}{2\pi(\alpha')^2}} \simeq \log \frac{2\pi(\alpha')^2 \Lambda^2}{\rho^2 e^{2i\theta}} ; \quad z \equiv \rho e^{i\theta} \quad (13)$$

and we have introduced a term corresponding to the bare coupling constant $g_{YM}(\Lambda)$. Eq.s (11) and (12) are identical to Eq.s (5) and (6).

3 The string construction of the “orientifold field theories”

In this section we summarize the string construction of the so called “orientifold field theories”². We consider the orientifold $\frac{0B}{\Omega' I_6(-1)^F_L}$ of type 0B string theory and we consider a D3 brane of this theory. Unlike 0B, it is a fully consistent theory with no tachyons and no R-R and NS-NS tadpoles. It can be shown that the gauge theory living on N D3 branes of this orientifold is a non-supersymmetric gauge theory with a gluon, six adjoint scalars and four Dirac fermions in the two-index (anti)symmetric representation of $SU(N)$. This gauge theory contains $8N^2$ bosonic and $8N(N \pm 1)$ fermionic degrees of freedom. Its one-loop β -function is equal to:

$$\beta(g_{YM}) = \frac{g_{YM}^3}{(4\pi)^2} \left[-\frac{11}{3}N + 6 \cdot \frac{N}{6} + 4 \cdot \frac{4}{3} \frac{N \pm 2}{2} \right] = \frac{g_{YM}^3}{(4\pi)^2} \left[0 \cdot N \pm \frac{16}{3} \right]. \quad (14)$$

For large N one recovers the Bose-Fermi degeneracy and the β -function vanishes as the one of $\mathcal{N} = 4$ super Yang-Mills. We call it “orientifold field theory” because it is non-supersymmetric, but it reduces for large N to a supersymmetric one, namely $\mathcal{N} = 4$ super Yang-Mills³.

One can extend the previous construction to a non-conformal theory by considering the orbifold $\mathbb{C}^2/\mathbb{Z}_2$ of the previous orientifold. The gauge theory living in the world-volume of N fractional D3 branes of this orbifold is a non-supersymmetric one consisting of one gluon, two adjoint scalars and two Dirac fermions transforming according the two index (anti)symmetric representation of $SU(N)$. This gauge theory contains $4N^2$ bosonic and $4N(N \pm 1)$ fermionic degrees of freedom. Its one-loop β -function is equal to:

$$\beta(g_{YM}) = \frac{g_{YM}^3}{(4\pi)^2} \left[-\frac{11}{3}N + 2 \cdot \frac{N}{6} + 2 \cdot \frac{4}{3} \frac{N \pm 2}{2} \right] = \frac{g_{YM}^3}{(4\pi)^2} \left[-2N \pm \frac{8}{3} \right]. \quad (15)$$

For large N one recovers the Bose-Fermi degeneracy and the β -function reduces to that of $\mathcal{N} = 2$ super Yang-Mills.

²See Ref. [11] for details, References therein and Ref. [12] for the original construction of tachyon free orientifolds of 0B theory. See also Ref. [13] for a review of the properties of these theories.

³This theory has been discussed in Ref. [14] and its gravity dual has been constructed in Ref. [15].

Finally one can consider the orbifold $\frac{C^3}{(Z_2 \times Z_2)}$ of the previous orientifold. The gauge theory living in the world-volume of N fractional D3 branes of this orbifold is a non-supersymmetric one consisting of one gluon and one Dirac fermion transforming according to the two-index (anti)symmetric representation of $SU(N)$. This gauge theory contains $2N^2$ bosonic and $2N(N \pm 1)$ fermionic degrees of freedom. Its one-loop β -function is equal to:

$$\beta(g_{YM}) = \frac{g_{YM}^3}{(4\pi)^2} \left[-\frac{11}{3}N + \frac{4}{3} \frac{N \pm 2}{2} \right] = \frac{g_{YM}^3}{(4\pi)^2} \left[-3N \pm \frac{4}{3} \right]. \quad (16)$$

For large N one recovers the Bose-Fermi degeneracy and the β -function reduces to that of $\mathcal{N} = 1$ super Yang-Mills. In conclusion we have constructed three systems of D3 branes that have in their world-volume non-supersymmetric gauge theories that in the large number of colours reduce to respectively $\mathcal{N} = 1, 2, 4$ super Yang-Mills.

4 Gauge/gravity correspondence for the “orientifold field theories”

Let us consider the interaction between a stack of N D3 branes of the orientifold $\frac{0B}{\Omega' I_6(-1)^{FL}}$ and a D3 brane dressed with an external constant gauge field:

$$Z^o = \int_0^\infty \frac{d\tau}{\tau} Tr_{NS-R} \left[\frac{e + \Omega' I_6}{2} \frac{1 + (-1)^{F_s}}{2} (-1)^{G_{bc}} P_{GSO} e^{-2\pi\tau L_0} \right]. \quad (17)$$

The term corresponding to e gives the annulus diagram, while the other term corresponds to the Möbius diagram. The annulus diagram does not give any contribution to the term quadratic in the gauge field as in $\mathcal{N} = 4$ super Yang-Mills. The Möbius diagram gives instead a non-vanishing contribution that, in the open-string channel, is equal to ($k = e^{-\pi\tau}$)

$$\frac{1}{g_{YM}^2} = \pm \frac{1}{(4\pi)^2} \int_0^\infty \frac{d\tau}{\tau} e^{-\frac{\rho^2\tau}{2\pi\alpha'}} \left(\frac{f_2(i\kappa)}{f_1(i\kappa)} \right)^8 \left[\frac{1}{3\tau^2} + k \frac{\partial}{\partial k} \log f_2^4(i\kappa) \right]. \quad (18)$$

The previous expression can be transformed in the closed-string channel by the transformation $\tau = \frac{1}{4t}$ ($q = e^{-\pi t}$):

$$\frac{1}{g_{YM}^2} = \pm \frac{1}{(4\pi)^2} \int_0^\infty \frac{dt}{4t^3} e^{-\frac{\rho^2}{8\pi\alpha't}} \left(\frac{f_2(iq)}{f_1(iq)} \right)^8 \left[\frac{4}{3} + \frac{1}{\pi} \partial_t \log f_2^4(iq) \right]. \quad (19)$$

They are finite both for $\tau \rightarrow \infty$ ($t \rightarrow 0$) and for $\tau \rightarrow 0$ ($t \rightarrow \infty$). Hence there are no divergences at the string level if $\rho \neq 0$. Otherwise infrared divergences appear. Performing the field-theory limit ($\alpha' \rightarrow 0$ and $\sigma \equiv 2\pi\alpha'\tau$ fixed) one gets UV divergences and must introduce a cut-off ($\mu = \frac{\rho}{2\pi\alpha'}$):

$$\frac{1}{g_{YM}^2(\mu)} = \frac{1}{g_{YM}^2(\Lambda)} \pm \frac{1}{3\pi^2} \int_{\mu^2/\Lambda^2}^\infty \frac{d\sigma}{\sigma} e^{-\sigma} = \frac{1}{g_{YM}^2(\Lambda)} \mp \frac{1}{3\pi^2} \log \frac{\mu^2}{\Lambda^2}, \quad (20)$$

that corresponds to the right β -function. The field theory limit in the closed-string channel ($t \rightarrow \infty$ and $\alpha' \rightarrow 0$ with $s = 2\pi\alpha't$ fixed) gives a vanishing contribution, NOT reproducing the correct β -function of the theory. This means that the gauge/gravity

correspondence works only in the large N limit, where the contribution of the Möbius strip is suppressed, the D3 branes are not interacting and the gauge theory recovers the Bose-Fermi degeneracy in the spectrum.

Let us consider the orbifolds C^2/\mathbb{Z}_2 of the previous orientifold. In this case we have two additional terms with respect to the previous orientifold, namely we get:

$$\int_0^\infty \frac{d\tau}{\tau} Tr_{\text{NS-R}} \left[\left(\frac{1+h}{2} \right) \left(\frac{e+\Omega' I_6}{2} \right) \left(\frac{1+(-1)^{F_s}}{2} \right) (-1)^{G_{bc}} P_{GSO} e^{-2\pi\tau L_0} \right]. \quad (21)$$

Extracting the quadratic term in the gauge field we get (in the field theory limit performed in the open string channel):

$$\frac{1}{g_{YM}^2} = \frac{1}{16\pi^2} \left[2N \mp \frac{8}{3} \right] \log \frac{\mu^2}{\Lambda^2} \quad (22)$$

where the first term comes from the twisted annulus diagram and the second one from the untwisted Möbius strip. Extracting the running coupling constant from the closed string channel we get again only the leading term in N . The twisted annulus and the twisted Möbius produce also a term proportional to the topological charge of the gauge theory, namely:

$$-\frac{i(N \pm 2)}{32\pi^2} \int d^4x F_{\alpha\beta}^a \tilde{F}^{a\alpha\beta} \int_0^\infty \frac{d\tau}{\tau} e^{-\frac{\rho^2\tau}{2\pi\alpha'}}.$$

From it we can extract the value of θ_{YM} as before getting:

$$\theta_{YM} = -i(N \pm 2) \int_0^\infty \frac{d\tau}{\tau} e^{-\frac{\rho^2\tau}{2\pi\alpha'}} \rightarrow -2\theta(N \pm 2). \quad (23)$$

Since the θ_{YM} -angle does not receive threshold corrections getting contribution only from the massless open string states that, under open/closed string duality, are transformed only into massless closed string states, it is clear that it can be equivalently obtained from either channels. The previous analysis can also be extended [11] to the orbifold $\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ obtaining similar results.

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